

## Kinematic Analysis of Human Leg

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**Abstract** - In present work kinematic analysis of the human leg determines the velocities and accelerations of the different links of the human leg viz. Femur and tibia. In this analysis, the velocity and acceleration diagrams for the human leg have been performed. Later by changing the link lengths in equal and unequal length ration has been performed for running on a treadmill device during stance phase. The parameters for the kinematic analysis are obtained for real time running which can be used for gait analysis of humanoid robots, selection of athletics etc. We can consider a human leg as a mechanism since it has relative motion in between the links. The major joints in a human leg mechanism are hip joint, knee joint and ankle joint. The hip joint joins' the femur and spine, knee joint joins femur and tibia, ankle joint joins' tibia and foot. In real life there are ball and socket joints between all these joints but in practical we can consider them as turning pairs. The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as angular velocities of the links. The velocity analysis is prerequisite for acceleration analysis which further leads to force analysis of various links of a mechanism. To facilitate such study, a mechanism is represented by a skeleton or a line diagram, commonly known as configuration diagram. Velocities and accelerations in machines can be determined either analytically or graphically. Graphical analysis is more direct and is accurate to an acceptable degree and thus cannot be neglected.

**Keywords:** Kinematic analysis, Biomechanics, Velocity analysis, Acceleration analysis.

### 1. INTRODUCTION

Biomechanics is concerned with the description of motion and how forces create motion. Forces acting on living things can create motion, be a healthy stimulus for growth and development, or overload tissues, causing injury. Biomechanics provides conceptual and mathematical tools that are necessary for understanding how living things move and how kinesiology professionals might improve movement or make movement safer. Biomechanics plays a crucial role in establishing the relation between forces involved in and the movement of links of a human being as well as in robotic applications for the present-day scenario of the technological world. The science involved in the human body in which muscles, bones, tendons and ligaments work together for the movement such as stair ascent, stair descent and walking. The intrinsic mechanics of the robot movement as well as human

beings gradually became clear through the work of the scientists in the specified fields of robotics and biomechanics respectively. Study of structure and function of biological systems of human beings is carried out by principles of classical mechanics. Mechanism of the movement of both human beings and robots is closely related to engineering, often uses traditional engineering science to analyze biological systems and to correlate these forces in order to replace human beings with robots. Srinivasan [1] has explained the process of gait cycle and the human leg movements during running or walking in both swing phase and stance phase. Godest et al. [2] has explained the approach of the forces during human gait cycle. Abbas et al. [3] investigated different parameters while running such as forces, angles at different positions of the hip and knee joints. Madeti et al. [4] has explained how to approach for static and dynamic analysis of knee joint. Komistek et al. [5] has explained how the ligaments of the knee act in different conditions. Erkin [6] has explained the detailed terminology and has performed the bio kinematic analysis on the human body.



Figure 1 Human posture while running

Biomechanics and robotics provide a wide range of intelligence in action [7], from human body to machine. Literature also provides deep connections between the human body and the synthetic, showing how the same principles apply to both, starting from sensing, through control, to actuation. Both fields can draw inspiration and better understanding from each other. This regeneration type perception [8] is particularly important at a period when service robotics is observed and so, as the next technological revolution. Humanoid robots are designed further in performing most of the activity's humans do.

## 2. HUMAN LEG MECHANICS AND PARAMETERS

The human leg can be divided into two parts as upper portion and lower portion. The upper portion of the leg starts with the longest and the largest bone of the human skeletal system that is called femur. The lower portion of the leg [9] continues with the two bones that are called tibia and fibula. Carrying the overall weight of the human body, tibia is the strongest bone of the skeletal system. There are two important joint formations in the system. The connection of the femurs nearly spherical head [10] with the hip bone results in the formation of the hip joint, while the connection of femurs lower extremity with the tibia forms the knee joint. The knee joint is protected and covered by a thick circular and triangular bone patella (knee cap or kneecap), which articulates with the femur. As the human legs are specifically adapted to the bipedal locomotion by the help of the location of the human body gravity center and stay under the heavy load of the human body, all the mentioned bones have special strength values, structures, and mobility's.

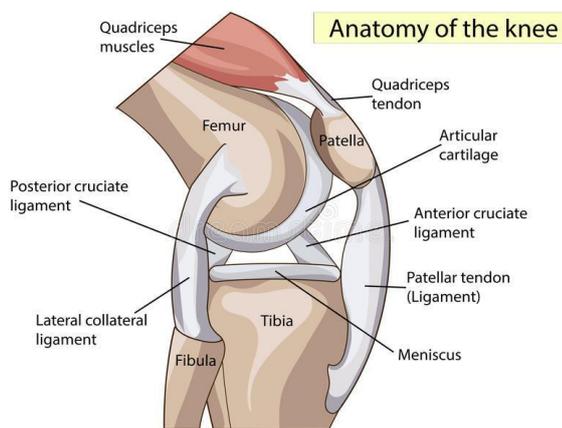


Figure 1 Anatomy of knee

### Kinematic Parameters

Lengths of the links:

- Length of tibia- 360mm to 400mm
- Length of femur- 420mm to 440mm
- Length of foot- 150 mm

Velocity of foot (speed of treadmill)- 1.38m/s

## 3. HUMAN LEG MODELLING

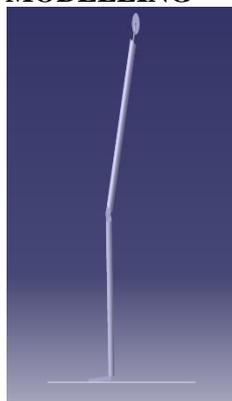


Figure 2 Human leg modelling

A CATIA model was drawn as equivalent to the human leg where it can be assumed that the human leg is running on a treadmill device with a speed of 1.38m/s, and the hip joint is considered as a fixed point and the foot has a speed equal to the speed of treadmill device. Thus the foot will acquire a velocity equal to that of the velocity of the treadmill. The model has a foot, femur, knee joint, tibia and a fixed hip joint.

## 4. ANALYSIS

Since this is kinematic analysis of the human leg, the velocity and acceleration analysis are performed during the stance phase where the foot remains in contact with the ground. The average lengths of the human bones are taken into consideration. That is the length of the femur is taken as 420mm, length of tibia taken as 360mm and length of foot is taken as 150mm. by taking these parameters into consideration, the velocity and acceleration diagrams have been drawn.

Now the problem occurs with the lengths of the links where the lengths of all people can't be the same. Hence the velocity and acceleration can't be the same for all people as the lengths of the links are different.

There are two cases where the lengths of the links vary. They are:

1. Unequal length ratio
2. Equal length ratio

$$\frac{\text{length of femur}}{\text{length of tibia}}$$

Where the length ratio=

### 4.1 Velocity and Acceleration of The Human Leg

Initially the velocity and acceleration diagrams are drawn for the human leg with lengths equal to the average length of the respective bones. And later by changing the lengths according to the length limit of the respective length of the bone the velocity and acceleration diagrams are drawn.

#### Taken parameters:

Length of femur = 420mm

Length of tibia = 360mm

Length of foot = 150mm

Angle made by femur with horizontal = 16

Angle between femur and tibia = 150

Speed of treadmill = velocity of foot = 1.38m/s

The skeleton form of the human leg can be represented as Figure 3:

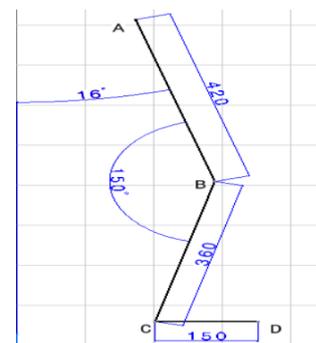


Figure 4 line representation of human leg

## 4.2 Procedure

### 4.2.1 Velocity analysis

The various joints can be represented with alphabets where A indicates the hip joint and it is a fixed joint, B indicates the knee joint, C indicates the ankle joint and D indicates the contact between foot and ground.

The link AB represents the femur

The link BC represents the tibia

The link CD represents the foot

Step 1: Make a point for the fixed points in the plane and mark them with small letters. Now name them as “a”, “d” as “a” is fixed hip and “d” is the ground.

Step 2: Draw a line parallel to the link CD with a length equal to the velocity of the link i.e., 138mm which is equivalent to 1.38m/s. Mark the other end as “c”

Step 3: Draw a line perpendicular to the link CB from “c” on both sides of it.

Step 4: Draw another line perpendicular to the link AB for “a”.

Step 5: Now join the two lines and the intersection point will be the point “b”.

The velocity diagram has been drawn where the length of the sides in the diagrams represents the velocity of the respective link.

Thus the obtained velocities of various links are:

$$ab = 66.77\text{mm} \approx V_{AB} \text{ (velocity of femur)} = 0.667\text{m/s}$$

$$bc = 76.08\text{mm} \approx V_{BC} \text{ (velocity of tibia)} = 0.768\text{m/s}$$

$$\text{The angular velocity of the tibia} = \frac{\text{velocity of tibia}}{\text{length of tibia}} = 0.20 \text{ rad/s}$$

$$\text{The angular velocity of femur} = \frac{\text{velocity of femur}}{\text{length of femur}} = 0.18 \text{ rad/s}$$

### 4.2.2 Acceleration analysis

Step 1: Take the pole point a1 or d1.

Step 2: Starting from d1, take the first vector d1c1.

Step 3: For the second vector, draw a line from c1 parallel to CB towards the point C from B and mark it as c1b'

Step 4: For the third vector, draw a line perpendicular to CB on both sides.

Step 5: For fourth vector, draw a line from a1 parallel to AB towards direction of B from A and mark it as a1b'

Step 6: For the fifth vector, draw a line perpendicular to AB.

Step 7: Mark the intersection point as b1.

Step 8: Join c1b1, a1b1.

$$\text{Step 9: } c1b1 = 78.26\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 7.826\text{m/s}^2$$

$$a1b1 = 48.84\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 4.884\text{m/s}^2$$

$$(4.88) \times (4.88)$$

$$\text{Step 10: } \alpha_{ab} = \frac{.420}{(7.826) \times (7.826)} = 56.79 \text{ rad/s}^2$$

$$\alpha_{bc} = \frac{.360}{.360} = 170.12 \text{ rad/s}^2$$

$$\alpha_{bc} = \frac{.360}{.360} = 170.12 \text{ rad/s}^2$$

Table 1 Vector table for acceleration analysis

S.	Vector	Magnitude(m/s <sup>2</sup> )	direction	sense
N				

1	c1d1	$\frac{(dc)+(dc)}{DC} = \frac{(1.38) \times (1.38)}{.150} = 12.69$	⊥DC	→D
2	c1b'	$\frac{(cb)+(cb)}{CB} = \frac{(0.768) \times (0.768)}{.360} = 1.03$	⊥BC	→C
3	a1b'	$\frac{(ab)+(ab)}{AB} = \frac{(0.667) \times (0.667)}{.420} = 1.60$	⊥AB	→B
4	c1b'b1	-	⊥BC	-
5	a1b'b1	-	⊥AB	-

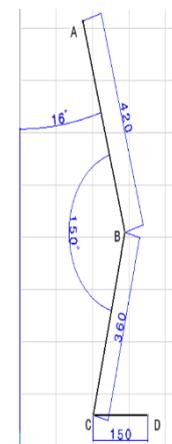


Figure 5 Velocity and acceleration diagrams for the human leg

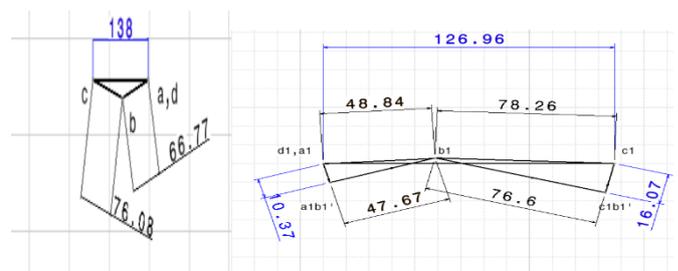


Figure 6 Velocity and acceleration diagram

## 5. VELOCITY ANALYSIS FOR UNEQUAL LENGTH RATIO

### Case 1:

Length of femur = 422mm

Length of tibia = 365mm

Length of foot = 150mm

Speed of treadmill = velocity of foot = 1.38m/s

By using the same steps as above, the obtained values are as below:

The obtained velocities of various links are:

$$ab = 66.33\text{mm} \approx V_{AB} \text{ (velocity of femur)} = 0.66\text{m/s}$$

$$bc = 77.08\text{mm} \approx V_{BC} \text{ (velocity of tibia)} = 0.77\text{m/s}.$$

The obtained accelerations of the links are:

$$c1b1 = 79.22\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 7.922\text{m/s}^2$$

$$a1b1 = 49\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 4.9\text{m/s}^2$$

#### Case 2:

$$\text{Length of femur} = 424\text{mm}$$

$$\text{Length of tibia} = 370\text{mm}$$

$$\text{Length of foot} = 150\text{mm}$$

$$\text{Speed of treadmill} = \text{velocity of foot} = 1.38\text{m/s}$$

By using the same steps as above, the obtained values are as below:

The obtained velocities of various links are:

$$ab = 67.3\text{mm} \approx V_{AB} \text{ (velocity of femur)} = 0.673\text{m/s}$$

$$bc = 78.4\text{mm} \approx V_{BC} \text{ (velocity of tibia)} = 0.784\text{m/s}$$

The obtained accelerations of the links are:

$$c1b1 = 80.39\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 8.039\text{m/s}^2$$

$$a1b1 = 49.26\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 4.926\text{m/s}^2$$

#### Case 3:

$$\text{Length of femur} = 426\text{mm}$$

$$\text{Length of tibia} = 375\text{mm}$$

$$\text{Length of foot} = 150\text{mm}$$

$$\text{Speed of treadmill} = \text{velocity of foot} = 1.38\text{m/s}$$

By using the same steps as above, the obtained values are as below:

The obtained velocities of various links are:

$$ab = 68.2\text{mm} \approx V_{AB} \text{ (velocity of femur)} = 0.682\text{m/s}$$

$$bc = 79.1\text{mm} \approx V_{BC} \text{ (velocity of tibia)} = 0.791\text{m/s}$$

The obtained accelerations of the links are:

$$c1b1 = 81.41\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 8.141\text{m/s}^2$$

$$a1b1 = 49.49\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 4.949\text{m/s}^2$$

#### Case 4:

$$\text{Length of femur} = 428\text{mm}$$

$$\text{Length of tibia} = 380\text{mm}$$

$$\text{Length of foot} = 150\text{mm}$$

$$\text{Speed of treadmill} = \text{velocity of foot} = 1.38\text{m/s}$$

By using the same steps as above, the obtained values are as below:

The obtained velocities of various links are:

$$ab = 69\text{mm} \approx V_{AB} \text{ (velocity of femur)} = 0.69\text{m/s}$$

$$bc = 80.2\text{mm} \approx V_{BC} \text{ (velocity of tibia)} = 0.802\text{m/s}$$

The obtained accelerations of the links are:

$$c1b1 = 82.58\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 8.258\text{m/s}^2$$

$$a1b1 = 49.82\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 4.982\text{m/s}^2$$

#### Case 5:

$$\text{Length of femur} = 430\text{mm}$$

$$\text{Length of tibia} = 385\text{mm}$$

$$\text{Length of foot} = 150\text{mm}$$

$$\text{Speed of treadmill} = \text{velocity of foot} = 1.38\text{m/s}$$

By using the same steps as above, the obtained values are as below:

The obtained velocities of various links are:

$$ab = 70.2\text{mm} \approx V_{AB} \text{ (velocity of femur)} = 0.702\text{m/s}$$

$$bc = 81.3\text{mm} \approx V_{BC} \text{ (velocity of tibia)} = 0.813\text{m/s}$$

The obtained accelerations of the links are:

$$c1b1 = 83.64\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 8.364\text{m/s}^2$$

$$a1b1 = 50.1\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 5.01\text{m/s}^2$$

#### Case 6:

$$\text{Length of femur} = 435\text{mm}$$

$$\text{Length of tibia} = 390\text{mm}$$

$$\text{Length of foot} = 150\text{mm}$$

$$\text{Speed of treadmill} = \text{velocity of foot} = 1.38\text{m/s}$$

By using the same steps as above, the obtained values are as below:

The obtained velocities of various links are:

$$ab = 70.9\text{mm} \approx V_{AB} \text{ (velocity of femur)} = 0.709\text{m/s}$$

$$bc = 82.3\text{mm} \approx V_{BC} \text{ (velocity of tibia)} = 0.823\text{m/s}$$

The obtained accelerations of the links are:

$$c1b1 = 84.67\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 8.467\text{m/s}^2$$

$$a1b1 = 50.58\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 5.058\text{m/s}^2$$

#### Case 7:

$$\text{Length of femur} = 422\text{mm}$$

$$\text{Length of tibia} = 365\text{mm}$$

$$\text{Length of foot} = 150\text{mm}$$

$$\text{Speed of treadmill} = \text{velocity of foot} = 1.38\text{m/s}$$

By using the same steps as above, the obtained values are as below:

The obtained velocities of various links are:

$$ab = 71.8\text{mm} \approx V_{AB} \text{ (velocity of femur)} = 0.718\text{m/s}$$

$$bc = 83.4\text{mm} \approx V_{BC} \text{ (velocity of tibia)} = 0.834\text{m/s}$$

The obtained accelerations of the links are:

$$c1b1 = 85.79\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 8.579\text{m/s}^2$$

$$a1b1 = 50.98\text{mm} \approx a_{AB} \text{ (acceleration of femur)} = 5.098\text{m/s}^2$$

**Case 8:**

Length of tibia = 400mm

Length of foot = 150mm

Speed of treadmill = velocity of foot = 1.38m/s

By using the same steps as above, the obtained values are as below:

The obtained velocities of various links are:

ab = 72.3mm  $\approx V_{AB}$  (velocity of femur) = 0.723m/s

bc = 84.1mm  $\approx V_{BC}$  (velocity of tibia) = 0.841m/s

Length of femur = 440mm

The obtained accelerations of the links are:

c1b1 = 86.9mm  $\approx a_{AB}$  (acceleration of femur) = 8.69m/s<sup>2</sup>

a1b1 = 51.31mm  $\approx a_{AB}$  (acceleration of femur) = 5.131m/s<sup>2</sup>

**Velocity analysis for equal length ratio**

When the length ratio is kept constant, the velocity diagrams do not change with respect to change in length. Thus, the velocity of each link becomes constant. Hence a person with a long step length and less step time can move forward at higher speed rate.

**6. RESULTS: TABULAR FORMS AND GRAPHS**

All the values obtained through the above analysis were tabulated and graphs are drawn. And the same procedure is done for different speeds of the treadmill.

Case 1: unequal length ratio

Table 2 Speed of the foot = 1.38 m/s

Sl.no	Tibia length (mm)	Femur length (mm)	Tibia velocity (m/s)	Femur velocity (m/s)	Tibia trans. Acc. (m/s <sup>2</sup> )	Tibia trans. Acc. (m/s <sup>2</sup> )	Tibia Acc. (m/s <sup>2</sup> )	Femur trans. Acc. (m/s <sup>2</sup> )	Femur rot. Acc. (m/s <sup>2</sup> )	Femur acc.(m/s <sup>2</sup> )
1	360	420	0.76	0.66	7.66	1.607	7.826	4.767	1.037	4.884
2	365	422	0.77	0.66	7.76	1.624	7.92	4.789	1.041	4.9
3	370	424	0.78	0.67	7.87	1.644	8.039	4.812	1.058	4.926
4	375	426	0.79	0.68	7.97	1.664	8.141	4.835	1.058	4.949
5	380	428	0.8	0.69	8.085	1.684	8.258	4.857	1.112	4.982
6	385	430	0.81	0.7	8.19	1.702	8.364	4.88	1.139	5.01
7	390	435	0.82	0.70	8.29	1.724	8.467	4.932	1.146	5.058
8	395	437	0.83	0.72	8.4	1.744	8.579	4.959	1.186	5.098
9	400	440	0.84	0.72	8.51	1.764	8.69	4.994	1.198	5.131

Table 3 Speed of the foot = 1.00 m/s

Sl.no	Tibia length (mm)	Femur length (mm)	Tibia velocity (m/s)	Femur velocity (m/s)	Tibia trans. Acc. (m/s <sup>2</sup> )	Tibia trans. Acc. (m/s <sup>2</sup> )	Tibia acc. (m/s <sup>2</sup> )	Femur trans. Acc. (m/s <sup>2</sup> )	Femur rot. Acc. (m/s <sup>2</sup> )	Femur Acc. (m/s <sup>2</sup> )
1	360	420	0.604	0.42	7.66	1.013	7.726	4.767	0.42	4.785
2	365	422	0.61	0.42	7.76	1.019	7.82	4.789	0.428	4.808
3	370	424	0.62	0.43	7.87	1.038	7.938	4.812	0.436	4.832
4	375	426	0.63	0.44	7.97	1.058	8.039	4.835	0.454	4.856
5	380	428	0.64	0.45	8.085	1.077	8.156	4.857	0.47	4.88
6	385	430	0.64	0.45	8.19	1.080	8.258	4.88	0.473	4.903
7	390	435	0.65	0.46	8.29	1.083	8.36	4.932	0.486	4.93
8	395	437	0.66	0.46	8.4	1.102	8.471	4.959	0.484	4.96
9	400	440	0.67	0.47	8.51	1.122	8.583	4.994	0.502	5.019

Table 4 Speed of the foot = 1.5m/s

Sl.no	Tibia length (mm)	Femur length (mm)	Tibia velocity (m/s)	Femur velocity (m/s)	Tibia trans. Acc. (m/s <sup>2</sup> )	Tibia rot. Acc. (m/s <sup>2</sup> )	Tibia Acc. (m/s <sup>2</sup> )	Femur trans. Acc. (m/s <sup>2</sup> )	Femur rot. Acc. (m/s <sup>2</sup> )	Femur Acc. (m/s <sup>2</sup> )
1	360	420	0.9	0.69	7.66	2.25	7.983	4.767	1.135	9.297
2	365	422	0.91	0.70	7.76	2.268	8.09	4.789	1.161	9.401
3	370	424	0.92	0.71	7.87	2.287	8.197	4.812	1.183	9.505
4	375	426	0.94	0.71	7.97	2.356	8.319	4.835	1.188	9.622
5	380	428	0.95	0.72	8.085	2.375	8.426	4.857	1.205	9.725
6	385	430	0.96	0.72	8.19	2.393	8.523	4.88	1.211	9.821
7	390	435	0.97	0.73	8.29	2.412	8.616	4.932	1.225	9.93
8	395	437	0.99	0.74	8.4	2.48	8.762	4.959	1.244	10.06
9	400	440	1	0.74	8.51	2.5	8.87	4.994	1.253	10.17

The various plots from the above data were plotted and they are as Figure 7:

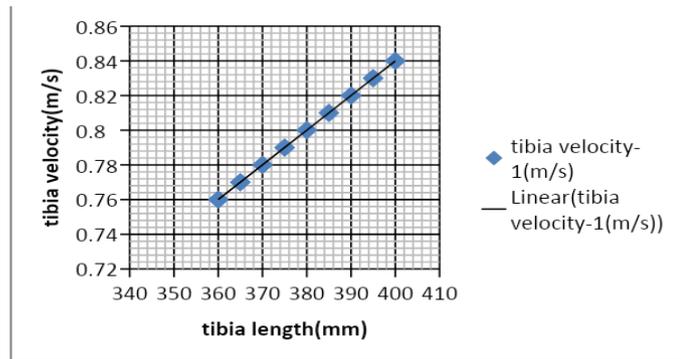


Figure 8 Length vs. velocity plot of tibia at speed 1

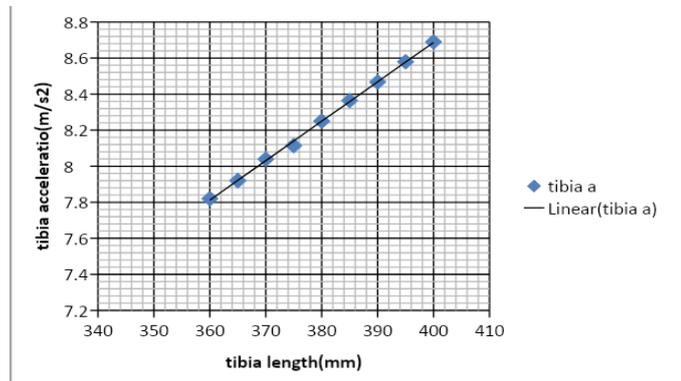


Figure 9 Length vs. acceleration plot of tibia at speed 1

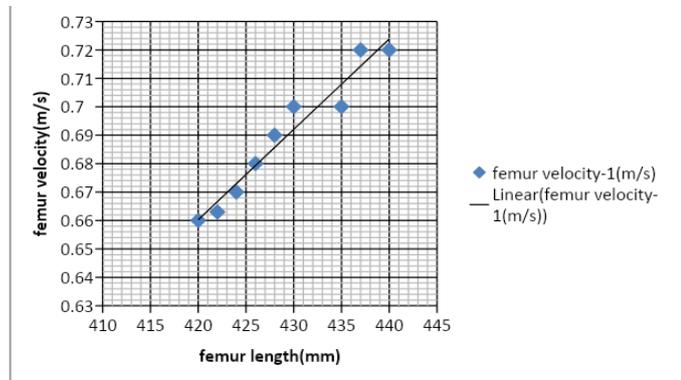


Figure 10 Length vs. velocity plot of femur at speed 1

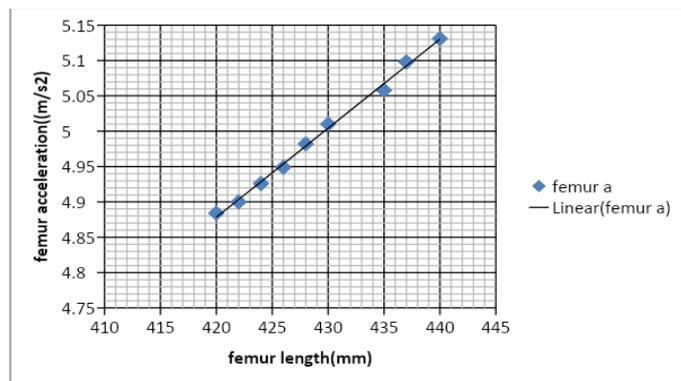


Figure 11 Length vs. acceleration plot of femur at speed 1

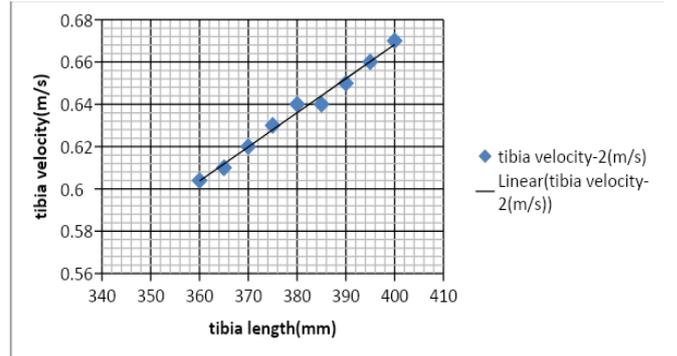


Figure 12 Length vs. velocity plot of tibia at speed 2

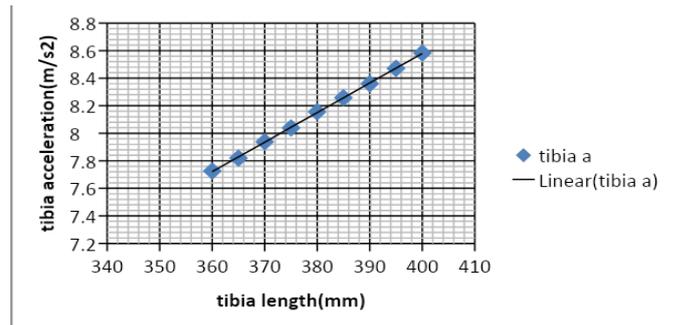


Figure 13 Length vs. acceleration plot of tibia at speed 2

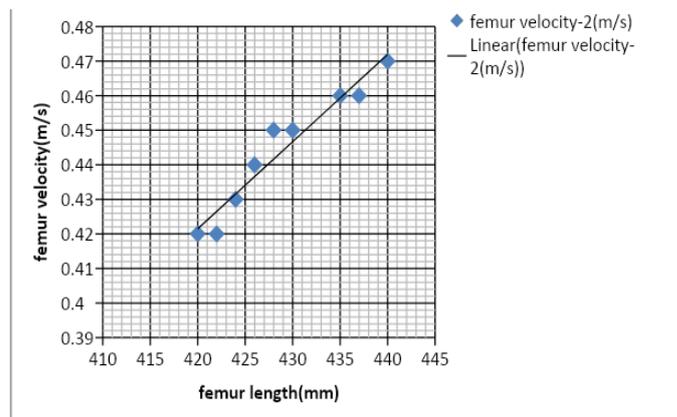


Figure 14 Length vs. velocity plot of femur at speed 2

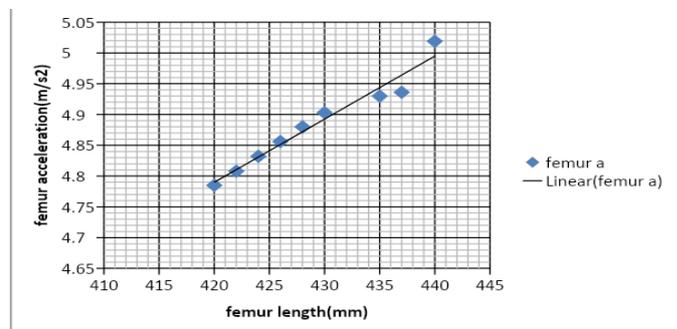


Figure 15 Length vs. acceleration plot of femur at speed 2

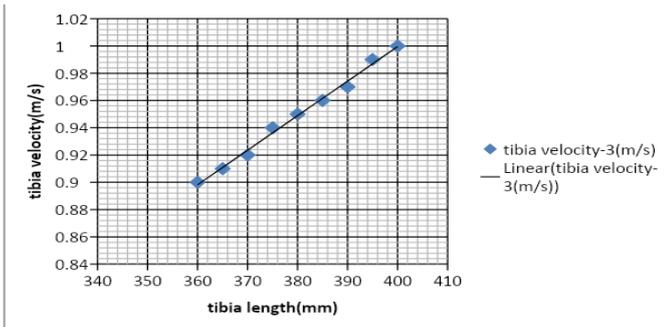


Figure 16 Length vs. velocity plot of tibia at speed 3

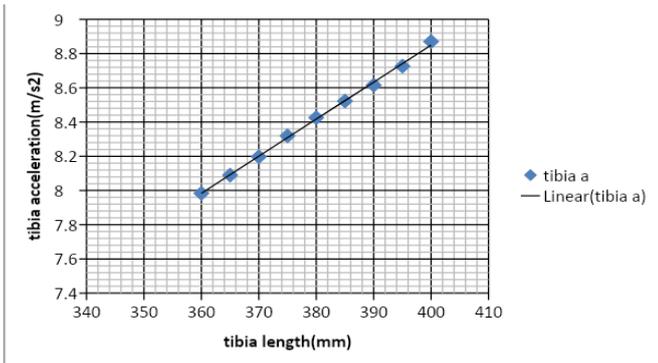


Figure 17 Length vs. acceleration plot of tibia at speed 3

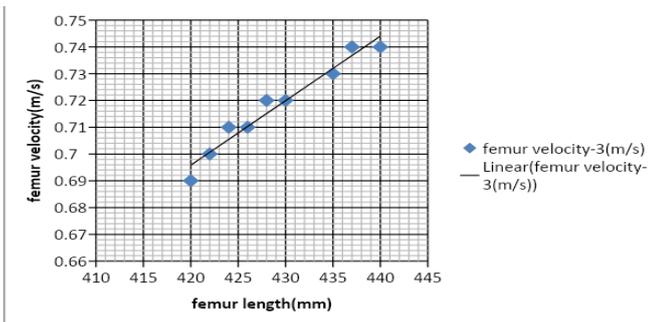


Figure 18 Length vs. velocity plot of femur at speed 3

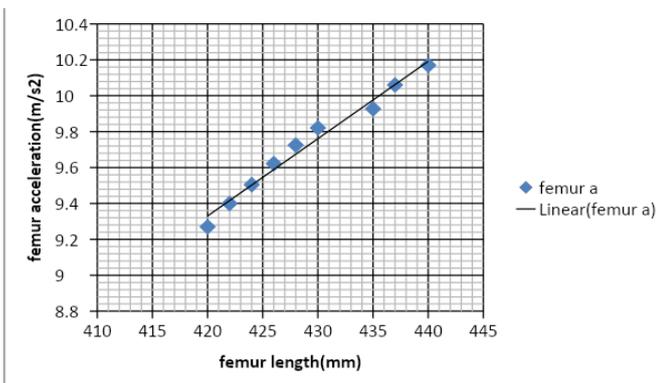


Figure 19 Length vs. acceleration plot of femur at speed 3

## 7. ANALYTICAL ANALYSIS

Also, from Eq. (2)

$$\beta = \frac{a \cos(\theta) - e}{b} \quad (6)$$

The analyses of the velocity and the acceleration depend upon the graphical approach and are suitable for finding out the velocity and the acceleration of the links of a mechanism in one or two positions of the crank. However, if it is required to find these values at various configurations of the mechanism or to find the maximum values of maximum velocity or acceleration [11-12], it is not convenient to draw velocity and acceleration diagrams again and again. In that case, analytical expressions for the displacement, velocity and acceleration in terms of the general parameters are derived. A desk-calculator or digital computer facilities the calculation work.

Figure 19 shows the human leg mechanism in which the foot does not pass through the axis. Angle  $\theta$  in clockwise from x-axis is taken as negative.

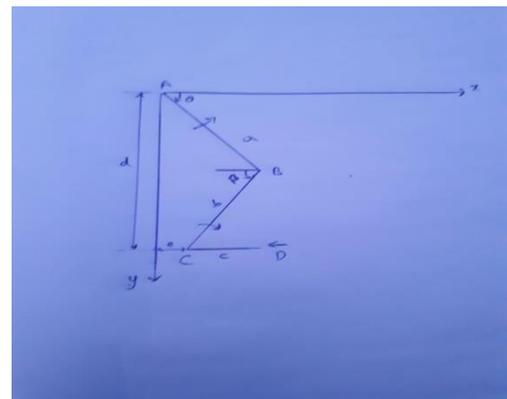


Figure 20 Human leg mechanism for analytical analysis [12]

Let  $e$  be the eccentricity,  $d$  be the distance from hip joint to ground.

Displacement along x-axis ,

$$a \cos(-\theta) + b \cos(\beta) + e = 0$$

$$b \cos(\beta) = a \cos(\theta) - e \quad (1)$$

displacement along y-axis,

$$a \sin(-\theta) + b \sin(\beta) + d = 0 \quad (2)$$

$$b \cos(\beta) = a \sin(\theta) - d \quad (3)$$

squaring Eqs.(1) and (2) and adding,

$$b^2 = e^2 + d^2 + a^2 - 2ae \cos \theta - 2ad \sin \theta$$

$$d^2 - (2a \sin \theta)d + a^2 - b^2 + e^2 - 2ae \cos \theta = 0$$

$$d^2 + C_1d + C_2 = 0 \quad (4)$$

where  $C_1 = -2a \sin \theta$

$$C_2 = a^2 - b^2 + e^2 - 2ae \cos \theta$$

Equation (4) is a quadratic in  $d$ . its two roots are,

$$d = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_2}}{2} \quad (5)$$

thus, if the parameters  $a$ ,  $b$ ,  $e$  and  $\theta$  of the mechanism are known, the output displacement can be computed.

### 7.1 Velocity Analysis

Differentiating the equations (1) and (2) with respect to time,

$$a\omega_a \sin \theta - b\omega_b \sin \beta + \dot{e} = 0 \quad (7)$$

$$-a\omega_a \cos \theta + b\omega_b \cos \beta = 0 \quad (8)$$

Multiplying Eq. (6) by  $\cos \beta$  and Eq. (7) by  $\sin \beta$  and adding,

$$a\omega_a(\sin \theta \cos \beta - \cos \theta \sin \beta) + \dot{e} \cos \beta = 0 \quad (9)$$

$$\dot{e} = \frac{a\omega_a \sin(\beta - \theta)}{\cos \beta} \quad (10)$$

from Eq. (7),

$$\omega_b = \frac{a\omega_a \cos \theta}{b \cos \beta} \quad (11)$$

$\omega_b$  provides the angular velocity of the tibia where as  $\dot{e}$  gives the linear velocity of the foot.

## 7.2 Acceleration Analysis

Differentiating the equations (4.6) and (4.7) with respect to time,

$$[a\alpha_a \sin \theta - a\omega_a^2 \cos \theta] - [b\alpha_b \sin \beta + b\omega_b^2 \cos \beta] + \ddot{e} = 0 \quad (12)$$

$$- [a\alpha_a \cos \theta - a\omega_a^2 \sin \theta] + [b\alpha_b \cos \beta - b\omega_b^2 \sin \beta] = 0 \quad (13)$$

Multiplying Eq. (12) by  $\cos \beta$  and Eq. (13) by  $\sin \beta$  and adding,

$$a\alpha_a \sin(\beta - \theta) - a\omega_a^2 \sin(\beta - \theta) - b\omega_b^2 \ddot{e} \cos \beta = 0 \quad (14)$$

$$\ddot{e} = \frac{a\alpha_a \sin(\beta - \theta) - a\omega_a^2 \sin(\beta - \theta) - b\omega_b^2}{\cos \beta} \quad (15)$$

$$\alpha_b = \frac{a\alpha_a \cos \theta - a\omega_a^2 \sin \theta - b\omega_b^2 \sin \beta}{b \cos \beta} \quad (16)$$

$\alpha_b$  provides the angular acceleration of the tibia where as  $\ddot{e}$  gives the linear acceleration of the foot.

## 8. CONCLUSIONS

The following conclusions are made from the present work

- Velocity analysis graphs can be used for gait analysis of humanoid robots and prosthesis
- The change in link lengths with equal length ratio doesn't alter the velocity of respective links whereas the step time and step distance cause the difference in speed of respective links.
- The change in link lengths with unequal length ratio alter the velocities of the respective links i.e. increase in link length results increase in velocity and vice versa
- This can be helpful for selecting the athletics for running
- In the similar way we can perform the analysis for various moving parts of the body such as hand while lifting weights, bowling, while swimming.
- This can be helpful for manufacturing the robot legs and artificial legs according to their requirements and lengths.
- From the velocity triangles it can be concluded that the velocities of the bones don't change when the length ratio is kept constant.

- When the length ratio is kept constant, the step length and step time are proportional to speed of links and a person with larger step length can move larger distance

- Thus, the person with larger lengths of tibia and femur can move fast.

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