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Thermo Elastic Analysis of Functionally Graded Rotating Annular Disk Profile using ANSYS

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Abstract: Functionally graded materials (FGMs) are characterized by their anisotropic nature, where physical properties vary continuously across dimensions either randomly or strategically to achieve specific characteristics. These materials exhibit overall properties that differ from the individual properties of their constituent parent materials. FGMs can be applied to metals, ceramics, and organic composites, leading to enhanced components that are increasingly valuable in various industries for maximizing the strength and integrity of engineered structures.

Although the processing of FGMs can be expensive, ongoing research in fabrication and processing techniques aims to reduce these costs, making the materials more accessible and applicable across a broader range of applications. This study provides a comprehensive overview of the diverse production techniques for manufacturing FGMs, detailing their characterization, advantages, and formulation, alongside recent advancements in the field.

In recent years, functionally graded shells have found extensive use in space vehicles, aircraft, nuclear power plants, and numerous other engineering applications. The stresses induced by centrifugal loads and internal or external pressures significantly impact their strength and safety. Therefore, the control and optimization of stress and displacement fields are crucial to minimizing the overall payload in industrial applications.

For this purpose, material modeling, geometric modeling, and finite element modeling of the disc are performed, followed by solving the numerical problem using the finite element software ANSYS. This approach helps in understanding the behavior of FGMs under various loading conditions and optimizing their design for better performance and safety in critical applications.

1. INTRODUCTION

Pure metals often fall short in engineering applications due to the need for properties that are inherently conflicting. For example, a material may need to be both hard and ductile, a combination that no single natural material can provide. To address this challenge, metals are combined with other metals or non-metals in a molten state, a process known as alloying. This process produces alloys with properties distinct from those of the parent materials.

An early and notable example of alloying is bronze, a combination of copper and tin, which marked a significant advancement in human history. However, the extent to which one material can dissolve into another in a molten solution is constrained by thermodynamic equilibrium limits. This limit dictates the solubility and the resultant properties of the alloy. At the point when greater amount of the alloying material is desired, then the traditional alloying cannot be used. Another constraint of traditional alloying is while alloying two unique materials with wide separated melting temperatures; it gets restrictive to consolidate these materials through this procedure. Powdered Metallurgy (PM) is another technique for creating part that can't be delivered through the conventional alloying, as alloys are produced in powdered form and a portion of the issues related with the ordinary alloying are overcome. In spite of the astounding attributes of powdered metallurgy, there exist a few impediments, which include: complex shapes and features that cannot be produced using PM, the parts are porous and have poor strength. In spite of the fact that these confinements are of favourable position to certain applications (e.g. filter and non-structural applications) but are detrimental to others. Another technique of producing materials with combination of properties is by combining materials in solid state, which is referred to as composite material.

Composite material are a class of cutting edge material, comprised of one or more materials combined in solid states with distinct physical and chemical properties. Composite material offers a superb combination of properties which are not the same from the individual parent materials and are also lighter in weight. Wood is a composite material from nature which consists of cellulose in a matrix of lignin. Composite materials will fail under outrageous working conditions through a process called delamination (separation of fibres from the matrix) [5]. This can occur for instance, in high temperature application where two metals with different coefficient of expansion are used. To take care of this issue, researchers in Japan in the mid 1980s, confronted with this challenge in a hypersonic space plane project requiring a thermal barrier (with outside temperature of 2000K and inside temperature of 1000K across less than 10 mm thickness), came up with a novel material called Functionally Graded Material (FGM).

1.1. Functionally graded materials (FGM):

These are multi-phase materials with graded properties. The degree of material properties in FGMs is accomplished by consistently changing the volume parts of the constituents. The subsequent material properties can be custom-made so that to fit the necessities presented in a large number of in fact requesting applications. FGMs are a class of cutting-edge composites made out of at least two discrete constituent stages with persistent and easily changing organization. These propelled materials with designable inclinations of piece can coordinate the benefits of constituent stages and show more grounded prevalence than homogeneous materials made of comparable constituents in applications. Regularly, the exact data of the shape and appropriation of particles may not be accessible, in this way the powerful material properties, viz. flexible moduli, shear moduli, thickness, and so forth of the reviewed composites are being assessed distinctly by the volume division conveyance and the estimated state of the scattered stage.

1.2. Areas of application of FGM

Some of the applications of functionally graded materials are highlighted below:

- Aerospace
- Medicine
- Defense
- Energy
- Optoelectronics
- Other areas of application:

*C*utting tool insert coating, automobile engine components, nuclear reactor components, turbine blade, heat exchanger, Tribology, sensors, fire retardant doors, etc. are the application fields of FGM. The rundown is perpetual and more application is jumping up as the handling innovation, cost of creation and properties of FGM improve.

2. LITERATURE REVIEW

The culmination of properties of all the forming materials of the FGM provide for a unique property of the freshly formed FG material having properties different from any parent material used. Thus, there is need to study about the Fabrication process of the FG material took off, to carry on for this **Rasheed at M. Mahamood et al** [1] in 2012, carried out a study on different processes of fabrication of the FG material. Since the FG materials are generally used in the form of surface coating, following for the cause of requirements a number of deposition techniques are studied under this which included vapour deposition technique, powder metallurgy, centrifugal method, solid Freeform (SFF) Fabrication method, Laser sintering technique etc. In the Vapour deposition technique the microstructure obtained is of fine quality but only thin layers of surface coatings can be done, also the nature of intensive energy consumption and production of poisonous gases makes it difficult for the general purpose use, also PM is good but the finesse of the material is compromised, it is good when a continuous structure is required. Although Centrifugal method provides for continuous grading but the shape forming is restricted to cylindrical structure. Solid Freeform technique uses CAD software for designing the material and thereby proving increase in production of Variable shapes, the speed of operation and manufacturing, the utilization of material is to the fullest and many more.

2.1. TECHNIQUES FOR SOLVING THE FGM PROBLEM

After the establishment of the need of the functionally graded material and the methods of manufacturing, a hefty task of calling for the formulation of the method of solving the variations in the properties with all types of attenuation of the loads and parameters came along the way. this is where **J.D. Claytona et al.** [3] in 2000, chipped away at the coordination of the element stiffness matrix for an axisymmetric finite element formulation, these two numerical integration strategies were introduced to beat the inaccuracy that emerged when an element was on or close to the axis of rotation.

FGM can take a number of shapes according to the need of application, it can be rectangular, square, circular etc. in the 2-D format and also cube, cuboid or any other free form shapes available, one such shape is Circular disk which finds its application in electronic as well as mechanical devices such as computer disks memory units, circular saws, turbine rotors etc.[5].

Generation of formulas and experimental as well as analytical studies were conducted to study the behaviour of disk under various conditions. Numerous approaches were used to find out the best way to reach out for the results. Some of them include the numerical approach, some went for the analytical approach, some for the Finite Element Analysis. In 2003 **Ahmet N. Eraslan et al.** [4] dealt with analytical solutions for the elastic–plastic stress distribution in rotating variable thickness annular disks obtained under plane stress assumption. The investigation depends on Tresca's yield criterion, its related flow rule and linear strain hardening material behaviour. The thickness of the disk is accepted to change in parabolic form in radial direction which leads to hyper geometric differential equations for the solution. It is demonstrated that, contingent upon the boundary conditions used, the plastic core may contain one, two or three different plastic regions governed by different mathematical forms of the yield criterion. The extension of these plastic regions with increasing angular velocity is acquired together with the distributions of stress, displacement and plastic strain. It is additionally indicated mathematically that in the limiting case the variable thickness disk solution reduces to the solution of rotating uniform thickness disk.

The analytical approach towards solving the problem is a commendable in its own ways but using two or three methods which gives same or accurate results defines for the authenticity of the work itself. **Ashraf M. Zenkour et al.** [5] in 2011found the exact analytical and numerical solutions for rotating variable-thickness annular disk. The internal and external edges of the rotating variable-thickness annular disk are considered to have free boundary conditions. Two different annular disks for the radially varying thickness are given.

The numerical Runge-Kutta solution as well as the exact analytical solution is available for the first disk while the exact analytical solution is not available for the second annular disk. Both exact and numerical results for stress function, stresses, strains and radial displacement were researched for the first annular disk of variable thickness. The exactness of the present numerical solution is discussed and its capacity of utilization for the second rotating variable-thickness annular disk is examined. At last, the distributions of stress function, displacement, strains, and stresses will be introduced. The appropriate examinations and conversations are made at the same angular velocity.

2.2. Objective

In an annular disc stress is maximum near the inner radius therefore, a comparative study of radial, circumferential and von mises stresses for different material, angular velocity and different radial thickness will be performed in order to find the suitability of the material to work in varying environment without failure.

A rotating annular disk of Al/ceramic functionally graded material [19] is validated. Properties of Al and ceramic are given in table 4.1. The property variation of the material in the FG disk along the radial direction is assumed to be power-law and Mori-Tanaka gradation (4.1.1), (4.1.2) and (4.1.3).

Table 1 Mechanical properties of Al and ceramic [19]

Material	E (GPa)	ρ (g/cm^3)	B (GPa)	G (GPa)
Al	71	2.70	58.333	26.9231
Ceramic	151	5.70	128.333	58.0769

Where E is modulus of elasticity, ρ is density, B is bulk modulus and G is shear modulus of the material.

Table 2 Mechanical properties of Al and Al_2O_3 [17]

Material	E (GPa)	ρ (g/cm^3)	α ($/ ^0C$)
Al	71	2.70	23.1×10^{-6}
Al_2O_3	380	0.96	8.0×10^{-6}

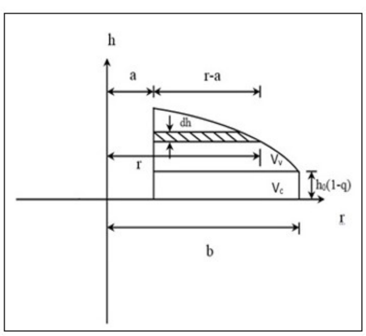

Fig.1.Half of the cross section of the variable thickness disk

3.Analysis Steps

Pre-processing: Material modelling, geometric modelling as well as Finite element modelling is done as a pre-processing step and boundary conditions are applied on the geometry.
Element selection: Plane182 type element is taken here which is used for 2-D modelling of solid structures. The element can be used as either a plane element (plane stress, plane strain or generalized plane strain) or an axisymmetric element. It is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions. Element behaviour is taken axisymmetric. Total 135 elements of 1 mm size is taken in radial direction.

Material modelling: For modelling of functionally graded material, 135 different materials are modelled having graded properties according to exponentially grading law and are assigned to each element.
Geometric modelling: In this stage geometry of the annular disk and various thickness profiles (convex, concave, linear and uniform) are modelled. 135 small areas of varying height and each of 1 mm width are generated and are glued to model the variable thickness of the disk.
Finite element modelling: In this stage the geometry of the disk is meshed and constrained boundary conditions, thermal boundary conditions and inertial boundary conditions are applied.

2. **Solution:** In this stage solution is done by using solution command solve.

3. **Post processing:** In this stage results are represented. Nodal displacement in radial direction and nodal stresses in radial as well as circumferential direction are evaluated.

3.1. Assumptions:

1. Material is continuous (free from internal cracks and voids).
2. Material is stress free (there is no initial or residual stress).
3. Material is functionally graded in one direction only (radial direction).
4. Mass is constant for all the thickness profile of the annular disk.
5. Poisson's ratio of the material is constant (v =0.3).
6. Young's modulus(E), density(ρ) and coefficient of linear expansion(α) of the material are assumed to be varying exponentially and are function of radius, r.
7. Temperature field is assumed to be varying exponentially
8. The analysis performed is linear elastic analysis.

4.RESULT AND DISCUSSION

The aim of this project is to make a comparative study of the radial displacements, radial stresses, circumferential stresses and von mises stresses developed in an annular disc made of functionally graded material of different thickness profile; when subjected to thermal load and an inertia force due to rotation of the disk.

4.1. Comparison of different thickness profiles:

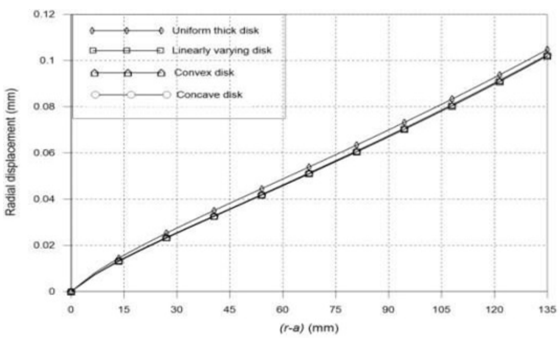

Fig 2. Comparison of radial displacement for fix-free boundary condition

Fig. 2 show the variation of radial displacement along the radial direction for different thickness profiles. For all thickness profiles, the value of radial displacement is zero at the inner radius and maximum at the outer radius, which confirms the constrained boundary condition applied on the model. Near the inner radius, in a very little zone, radial displacement variation curves has higher slope (from inner radius to 22 mm radius approximate), then after this zone, slope slightly decrease and remains approximate constant till the outer radius. From the comparison of radial displacements for all thickness profiles, it is observed that thickness of the disk has a very little effect on the radial displacement. Uniform thickness disk has a little higher radial displacement value as compared to other variable thickness disks.

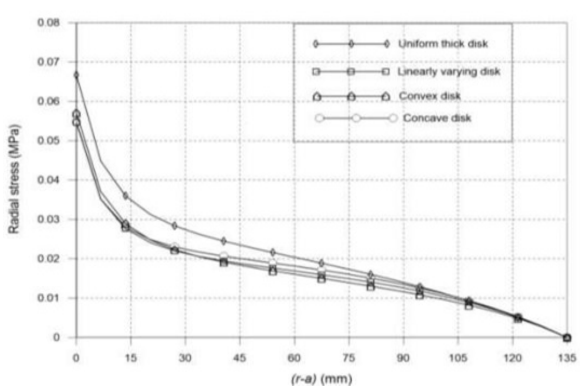

Fig 3 Comparison of radial stress for fix-free boundary condition

Fig. 3 show the variation of the radial stress along the radius for different thickness profiles. The value of radial stress is maximum at the inner radius and minimum that is zero at the outer radius for all thickness profiles, which is due to fix-free boundary condition applied on the disk. The value of maximum radial stress for uniform thickness disk is 0.068 MPa, for convex disk its value is 0.057 MPa and for linear and concave disk its value is 0.054 MPa.

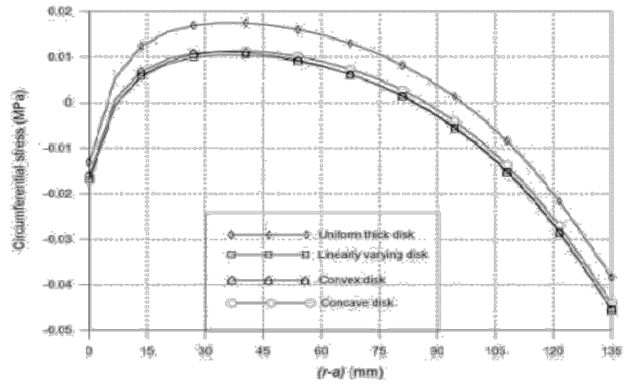

Fig 4 Comparison of circumferential stress for fix-free boundary condition

Fig. 4 show the variation of the circumferential stress along the radius for different thickness profiles. Circumferential stress induced in the disks are tensile and compressive both in nature. Magnitude of the compressive stress is higher than the tensile stress. Compressive stress is maximum at outer radius while tensile stress is maximum at approximate 50 mm radius for all thickness profiles. The value of maximum compressive stress is 0.038 MPa for uniform disk, 0.043 MPa for concave disk, 0.045 MPa for linear disk and convex disk. The value of maximum tensile stress is 0.018 MPa for uniform disk, 0.01 MPa for concave disk, 0.009 MPa for linear disk and convex disk.

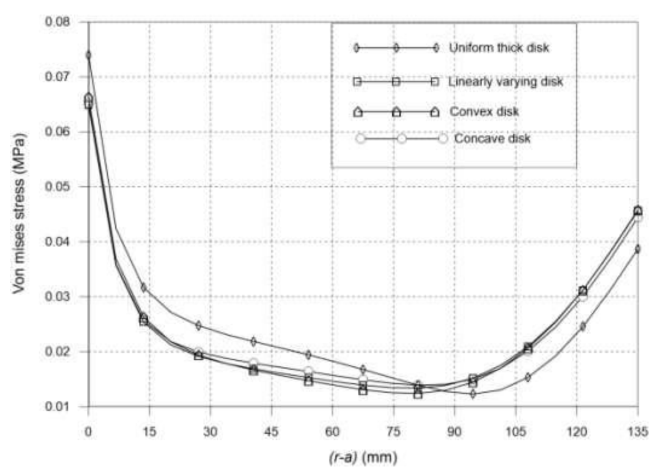

Fig 5 Comparison of von mises stress for fix-free boundary condition

Fig. 5. show the variation of the von mises stress along the radius for different thickness profiles. Von mises stress induced in the disks is completely tensile in nature. It is maximum at the inner radius for all the thickness profiles. The value of maximum stress is 0.075 MPa for uniform disk, 0.066 MPa for convex

disk, 0.065 MPa for linear disk and for concave disk. This is because linear disk and concave disk have more thickness at the root as compared to convex disk for constant mass.

The von mises stress variation curves may be divided into three zones, first from inner radius to 22 mm radius, where all the variable thickness disks have approximate same values except inner radius, then second from 25 mm to 85 mm in which convex thickness disk has minimum stress as compared to all other variable thickness disk and then from 85 mm to outer radius. In both the region 1 and 2 uniform thickness disk has the highest stress as compared to variable thickness disk but in region 3 uniform thickness disk has lowest stress as compared to variable thickness disk because uniform disk has higher thickness in this zone as compared to all other variable thickness.

4.2. Comparison of different ceramic materials in FGM

Alumina, zirconia and titanium carbide are the three ceramic materials considered of FGM with aluminum

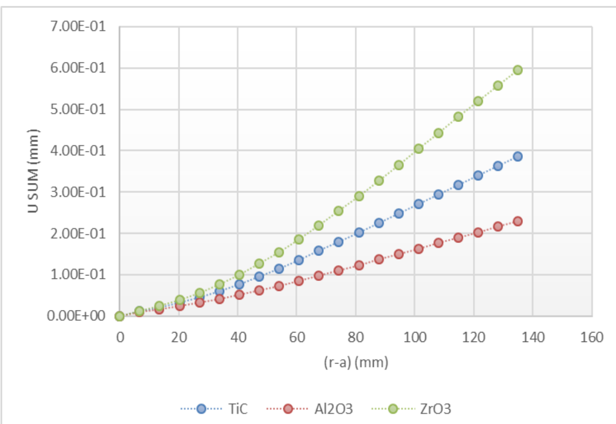

Fig 6. Comparison of total radial displacement for fix-free boundary condition

Since, alumina is showing the least deformation of all the above ceramic materials when made FGM with aluminum therefore alumina can be chosen as the material for FGM with aluminum.

4.3 Comparison of different angular velocity in FGM

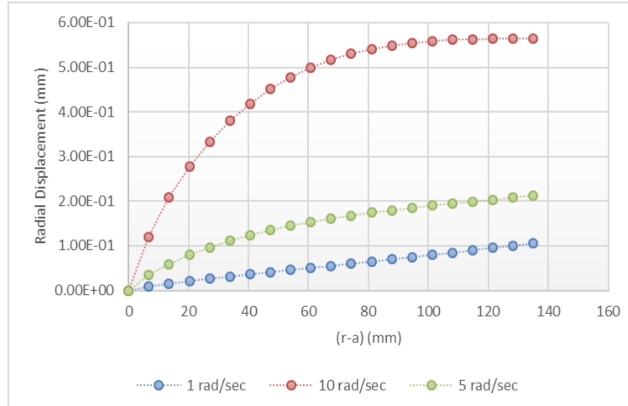

Fig 5.6: Comparison of radial displacement for fix-free boundary condition for AL-Alumina

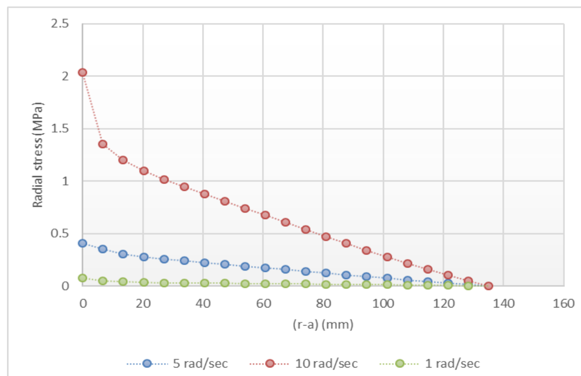

Fig 7. Comparison of radial stress for fix-free boundary condition for AL-Alumina

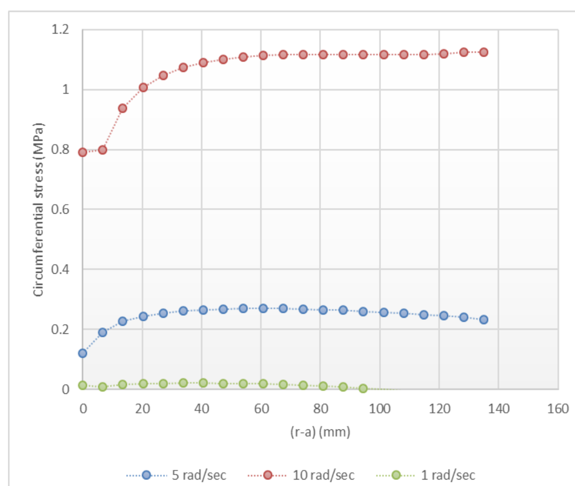

Fig 8. Comparison of circumferential stress for fix-free boundary condition for AL-Alumina.

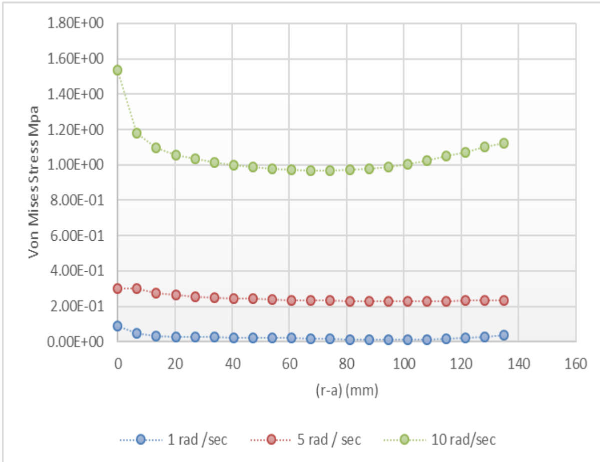

Fig 9. Comparison of von mises stress for fix-free boundary condition

Figure 5.6 demonstrates that radial displacement increases with rising angular velocity, while Figure 5.7 shows that radial stress also intensifies as angular speed increases. Notably, the increase in radial stress magnitude with higher angular speeds is more pronounced near the inner surface of the disk. Additionally, Figure 5.8 illustrates that circumferential stress escalates with increasing angular velocity, and Figure 5.9 indicates that von Mises stress also rises with higher angular speeds.
Conclusions
In this project, a rotating disk made of functionally graded material (FGM) is analyzed. The disk is subjected to exponentially varying material properties, mechanical body loads in the form of centrifugal forces, and thermal body loads due to uneven temperature distribution. Four types of thickness profiles are considered for a fix-free boundary condition: uniform thickness, linearly varying thickness, concave thickness, and convex thickness. Finite element modeling is performed, and the numerical analysis is conducted using ANSYS Mechanical APDL to compare the different thickness profiles. The conclusions drawn from the results are:
For all types of thickness profiles, a disk with uniform thickness exhibits higher displacement and stresses compared to disks with variable thickness under fix-free boundary conditions.
It is also observed that radial displacement is less influenced by thickness variations compared to stresses.
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5. Conclusions

In this project work rotating disk made up of functionally graded material is analysed. The disk is subjected to exponentially varying material properties and subjected to mechanical body load in the form of centrifugal force as well as thermal body load in the form of uneven temperature distribution. Four types of thickness profiles namely uniform thickness, linearly varying thickness, concave thickness and convex thickness is considered for fix-free boundary condition. Finite element modelling is done and numerical problem is analysed by the help of ANSYS Mechanical APDL and a comparison is made for all thickness profiles. The results obtained may be concluded as:

1.In all types of thickness profile, uniform thickness disk has a higher displacement and stresses as compared to variable thickness disk for the fix-free boundary condition.

2.Also it is observed that the radial displacement is less affected by the thickness variation as compared to stresses.

3.Von mises stress obtained is maximum near the inner radius zone for fix-free boundary condition. On the basis of which it is suggested that the disk should have higher strength in this region, means thickness, density and modulus of elasticity should be as high as possible in this region.

4.In comparison of different materials alumina is found to have the least deformation therefore functionally graded material best combination with aluminum out of zirconia, titanium carbide and alumina is alumina.

5.It is also seen that with gradual increase of angular velocity radial displacement also increases followed by radial stress, circumferential stress and von mises stress, therefore, the more the angular velocity more stresses will be induced and as a result radial displacement will also be higher.

5.1. Scope of further work:

1. Analysis of rotating disk made up of different types of FGM such as sigmoid low FGM, power law FGM, FGM made by mori tanaka etc.

2.Analysis of rotating disks made up of two dimensional or three dimensional FGM. Analysis of other thickness profile apart from parabolic concave and parabolic convex.

3. Thermo elastic analysis of rotating disks in different types of temperature field apart from exponential field.

4.Analysis of the Rotating disk for different boundary conditions. Dynamic analysis of the FGM rotating disks.

REFERENCES

[1] Rasheed M. Mahamood, Esther T. Akinlabi, Mukul Shukla and Sisa Pityana, 2012. Functionally Graded Material: An Overview. Proceedings of the World Congress on Engineering, Vol III.

[2] Zahra Sharif Khodaei, 2005. Preliminaries to Modeling and Analysis of Functionally Graded Materials. Czech Technical University in Prague.

[3] J.D. Claytona, J.J. Rencisb, 2000. Numerical integration in the axisymmetric finite element formulation. Advances in Engineering Software, 31: 137–141.

[4] Ahmet N. Eraslan, 2003. Elastic–plastic deformations of rotating variable thickness annular disks with free, pressurized and radially constrained boundary conditions. IJMS, 45: 643–667.

[5] Ashraf M. Zenkour, Daoud S. Mashat, 2011. Stress Function of a Rotating Variable-Thickness Annular Disk Using Exact and Numerical Methods. SCIRP Engineering, 3: 422-430.

[6] J. N. Sharma, Dinkar Sharma, Sheo Kumar 2011. Analysis of Stresses and Strains in a Rotating Homogeneous Thermoelastic Circular Disk by using Finite Element Method. International Journal of Computer Applications, Volume 35– No.13 (0975 – 8887).

[7] Hasan Callioglu, 2011. Stress analysis in a functionally graded disc under mechanical loads and a steady state temperature distribution. Sadhana Vol. 36, Part 1: 53–64.

[8] Maruthi B H, M. Venkatarama Reddy, K. Channakeshavalu, 2012. Finite Element Formulation for Prediction of Over-speed and burst-margin limits in Aero-engine disc. International Journal of Soft Computing and Engineering (IJSCE), Volume-2, Issue-3.

[9] M. Shariyat, R.Mohammadjani, 2013. Three-dimensional compatible finite element stress analysis of spinning two-directional FGM annular plates and disks with load and elastic foundation non uniformities. LAJSS, 10: 859 – 890.

[10] Mohammad Zamani Nejad, Mehdi Jabbari, Mehdi Ghannad, 2015. Elastic analysis of FGM rotating thick truncated conical shells with axially-varying properties under non-uniform pressure loading. Composite Structures, 122: 561–569.

[11] J N. Sharma, Dinkar Sharma, Sheo Kumar, 2012. Stress and strain analysis of rotating FGM Thermo elastic circular disk by using FEM. International Journal of Pure and Applied Mathematics, Volume 74 No. 3: 339-352.

[12] Abdur Rosyid, Mahir Es-Saheb, Faycal Ben Yahia, 2014. Stress Analysis of Nonhomogeneous Rotating Disc with Arbitrarily Variable Thickness Using Finite Element Method. Research Journal of Applied Sciences, Engineering and Technology, 7(15): 3114-3125.

[13] Nguyen Dinh Duc, Vu Thi Thuy Anh, Pham Hong Cong, 2014. Nonlinear axisymmetric response of FGM shallow spherical shells on elastic foundations under uniform external pressure and temperature. European Journal of Mechanics A/Solids, 45: 80-89.

[14] Minwoo Hong, Ilwook Park, Usik Lee, 2014. Dynamics and waves characteristics of the FGM axial bars by using spectral element method. Composite Structures, 107: 585–593.

[15] Manish Bhandari, Dr. Kamlesh Purohit, 2014. Analysis of Functionally Graded Material Plate under Transverse Load for Various Boundary Conditions. IOSR-Journal of Mechanical and Civil Engineering, Volume 10, Issue 5: 46-55.

[16] Hassan Zafarmand, Mehran Kadkhodayan, 2015. Nonlinear analysis of functionally graded nanocomposite rotating thick disks with variable thickness reinforced with carbon nanotubes. Aerospace Science and Technology, 41: 47–54.

[17] A.M. Afsar, J. Go, 2010. Finite element analysis of thermoelastic field in a rotating FGM circular disk. Applied Mathematical Modelling, 34: 3309–3320.

[18] P. Seshu, 2003. A text book of finite element analysis. PHI Learning Pvt. Ltd. Ch 5, pp 167-171.

[19] M. Bayat, B. B. Sahari, M. Saleem, E. Dezvareh, A. H. Mohazzab, 2011. Analysis of Functionally Graded Rotating Disks with Parabolic Concave Thickness Applying an Exponential Function and the Mori-Tanaka Scheme. IOP Conf. Series: Materials Science and Engineering, vol: (17).

[20] Mehdi Bayat, M. Saleem, B.B. Sahari, A.M.S. Hamouda and E. Mahdi, 2009. Mechanical and thermal stresses in a functionally graded rotating disk with variable thickness due to radially symmetry loads. International Journal of Pressure Vessels and Piping, 86: 357–372.

[21] Hasan Callioglu, Numan Behlul Bektas and Metin Sayer, 2011. Stress analysis of functionally graded rotating discs: analytical and numerical solutions. Acta Mech. Sin. 27(6):950–955.

[22] Hasan Callioglu, Numan Behlul Bektas and Metin Sayer, 2011. Stress analysis of functionally graded rotating discs: analytical and numerical solutions. Acta Mech. Sin. 27(6):950–955

[23] Hassan Zafarmand and Behrooz Hassani, 2014. Analysis of two-dimensional functionally graded rotating thick disks with variable thickness. Acta Mech 225, 453–464.

[24] Hasan Callioglu, Metin Sayer, Ersin Demir, 2011. Stress analysis of functionally graded discs under mechanical and thermal loads. Indian Journal of Engineering & Material Sciences, 18:111-118.

[25] Amani Nejad, Majid Abedi, Mohammad Hassan and Mehdi Ghannad, 2013. Elastic analysis of exponential FGM disks subjected to internal and external pressure, Central European Journal of Engineering 3(3): 459-465.